Knowledge in Quantum Mechanics

Vishal Johnson

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Introduction

Knowledge is an umbrella term used to indicate many sorts of correlations. As stated in the Stanford Encyclopaedia of Philosophy[1], there are at least 3 sorts of knowledge. It means slightly different things to know a person, to know how and to know a fact. This article is restricted to knowledge of facts. The purpose of this paper is to offer a quantum-esque exposition of knowledge.

Intuitively, knowledge is a mental entity that correlates with some "external fact". For the purpose of the paper let it be supposed that knowledge is possessed if

1. it is true,
2. the subject possessing the knowledge is certain of it.

So, knowledge is possessed by a subject who is sure[1], with probability 1, that a true fact obtains.

Let, $|\psi\rangle_c$ represent the state of liveliness of a cat. $|0\rangle_c$ represent the cat being dead, whereas, $|1\rangle_c$ represent the cat being alive. Let $|\psi\rangle_k$ represent the state of Schrödinger's knowledge of the cat's liveliness. $|0\rangle_k$ represents Schrödinger's knowledge that the cat is dead and $|1\rangle_k$ represents Schrödinger's knowledge that the cat is alive[2].

By the very definition of knowledge, it cannot be false. And thus, the liveliness state and knowledge state must correlate perfectly. So, the only valid states are:

$$\begin{aligned} &|0\rangle_c \otimes |0\rangle_k \,, \text{ and,} \\ &|1\rangle_c \otimes |1\rangle_k \,. \end{aligned} \tag{1}$$

[1]This surety could be used as a placeholder for justified belief.

[2]The state of knowledge of liveliness is no doubt a mental state. What kind of state is the state of liveliness of a cat? For something to be living or dead is for it to agree with a definition of life. It must, thus, also be a mental state ; but a mental state that is formed through agreement with an entire set of people, a societal state perhaps.

Entanglement and Mixed States

Schrödinger ascertains the liveliness of his cat and puts it in a box[3]. As there is no means of knowing whether the cat is alive or dead there is no knowledge possessed about the liveliness of the cat. In what state is the cat then, and knowledge of the cat?

The cat has a certain probability of being alive and this probability is a function of time. If the cat has been in the box for 1 hour, the probability of it being alive is quite high. On the other hand, if the cat has been in the box for 2 weeks, the probability of it being alive is quite low. So what is needed is a function that gives the probability of the cat being alive as a function of time. This, of course, is the hamiltonian of the system and for any real cat it would be very hard indeed to come up with a usable hamiltonian.

The combined state $|\psi\rangle_c \otimes |\psi\rangle_k$ is given by,

$$|\psi\rangle_c \otimes |\psi\rangle_k = \sqrt{p(t)}\,|0\rangle_c \otimes |0\rangle_k + \sqrt{1-p(t)}\,|1\rangle_c \otimes |1\rangle_k . \tag{2}$$

It is an entangled state. Knowledge of something implies entanglement with it.

Everything except the previous equation 2 could be consistently studied using classical analysis. Superposition states and entanglement are distinctly quantum phenomena and it would be instructive to delineate on these ideas[4].

What makes quantum states quantum is the possibility of being in superposition states. Whereas a classical cat can only be dead or alive, a quantum cat could be in a superposition of such states. It is in a sense alive and dead at the same time. The cat would be in this state until it is "measured", someone peeks inside the box. Once it is measured, the state of the cat is either dead or alive and it becomes classical again. There is a huge unresolved debate about this matter which brings together philosophy, physics and mathematics but this is not something that this paper does or even need to get into.

$$\begin{aligned} &\text{classical states: } \boxed{0} \text{ or } \boxed{1} \\ &\text{quantum states: } |0\rangle, |1\rangle \text{ or even } \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\ ^{5} \end{aligned} \tag{3}$$

What about entanglement? To study entanglement one must first study the combined states of multiple objects. The "cat" and "knowledge" states of before can be used for this purpose. The combined states of the liveliness and knowledge of the cat can be represented as in equation 1. However, an entangled

[3] In this case an ordinary box would not do, the purpose is to completely isolate the cat from Schrödinger so that there is no way for energy to be exchanged between them.

[4] Only the very basics of quantum states would be explored here, the reader is implored to refer to any treatment of quantum information theory many of which can be found on the internet. A great book for quantum computing is Nielsen and Chuang[2].

[5] The probability of measuring the state $|0\rangle$ is $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$ and so equivalently is the probability of measuring $|1\rangle$. And as the probability of obtaining some state is 1, it must be the case that the coefficients of a superposition state are such that their squares add to 1.

state is something quite different. To illustrate this consider a non entangled combined state in which the cat and knowledge states are both superposition states.

$$\begin{aligned}
|\psi\rangle_c &= \frac{1}{\sqrt{2}}|0\rangle_c + \frac{1}{\sqrt{2}}|1\rangle_c \\
|\psi\rangle_k &= \frac{1}{\sqrt{2}}|0\rangle_k + \frac{1}{\sqrt{2}}|1\rangle_k \\
\implies |\psi\rangle_c \otimes |\psi\rangle_k &= (\frac{1}{\sqrt{2}}|0\rangle_c + \frac{1}{\sqrt{2}}|1\rangle_c) \otimes (\frac{1}{\sqrt{2}}|0\rangle_k + \frac{1}{\sqrt{2}}|1\rangle_k) \\
&= \frac{1}{2}|0\rangle_c \otimes |0\rangle_k + \frac{1}{2}|0\rangle_c \otimes |1\rangle_k + \frac{1}{2}|1\rangle_c \otimes |0\rangle_k + \frac{1}{2}|1\rangle_c \otimes |1\rangle_k
\end{aligned} \tag{4}$$

These states are separable, meaning that by looking at the combined state one can find out the states of cat and knowledge. On the other hand, the state of equation 2 is an entangled state. It is not possible to separate into the state of cat and state of knowledge but the state simply is the combined state of both. Cat and knowledge are forced to take on the same values. The claim of this section is that knowledge is an entangled state.

So what does Schrödinger possess when the cat is in the box? He does not possess knowledge but has a certain belief about the existence of the cat. Let it be suppossed that Schrödinger is aware of the hamiltonian of liveliness of the cat and that he can predict the probability of the cat being alive at any point of time.

What Schrödinger possesses is a mixed state:

$$\begin{aligned}
&|0\rangle_k, \text{ with probability } p(t), \\
&|1\rangle_k, \text{ with probability } 1 - p(t).
\end{aligned} \tag{5}$$

Thus, belief states are mixed states[6]!

These states can be equally well represented using classical states as using quantum states.

$$\begin{aligned}
&\boxed{0}_k, \text{ with probability } p(t), \\
&\boxed{1}_k, \text{ with probability } 1 - p(t).
\end{aligned} \tag{6}$$

So, the very motivation to use quantum states to represent knowledge is that quantum states capture the idea of entanglement so very well.

An Example

Now, for a specific example. Consider the cat to be well and alive at the beginning of the experiment. It is in the state:

[6] A mixed state is distinctly different from a superposition state. The "quantum"-ness of superposition is absent in a mixed state.[2]

$$|\psi\rangle_c \otimes |\psi\rangle_k = |0\rangle_c \otimes |0\rangle_k \,. \tag{7}$$

How can we develop a toy hamiltonian? Consider the function $\frac{\arctan}{2} + \frac{\pi}{4}$; its value varies from 0 to $\frac{\pi}{2}$ and thus the cosine and sine of its values would vary from 0 to 1. Now the state,

$$\begin{aligned} |\psi\rangle_c \otimes |\psi\rangle_k = &\cos\left(\frac{\arctan(t-15)}{2} + \frac{\pi}{4}\right) |0\rangle_c \otimes |0\rangle_k + \\ &\sin\left(\frac{\arctan(t-15)}{2} + \frac{\pi}{4}\right) |1\rangle_c \otimes |1\rangle_k \end{aligned} \tag{8}$$

would vary between having a high probability of being $|0\rangle_c \otimes |0\rangle_k$ to a high probability of being $|1\rangle_c \otimes |1\rangle_k$ at around 15 ± 2 days[7].

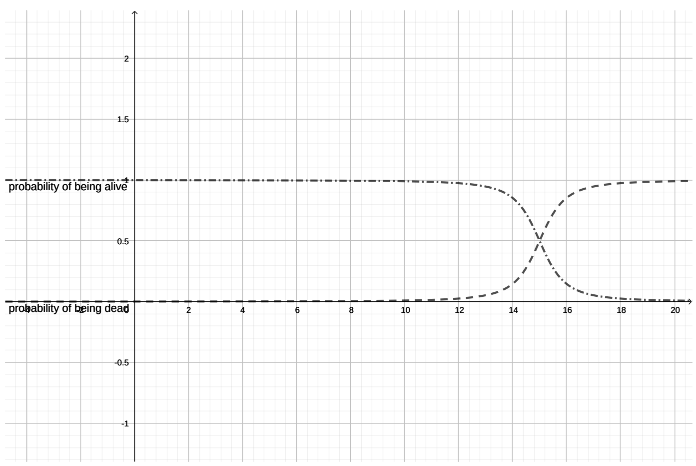

Figure 1: The probabilities of being alive and dead as a function of time as per equation 8.

As it can be seen, the state is automatically normalised and seems like a reasonable model for the "liveliness of a cat" hamiltonian.

Consider now a valve that connects to a container of poison gas. If the valve is opened, the cat would almost surely die within a few hours. This hamiltonian can easily be obtained by modifying the one in equation 8.

$$\begin{aligned} |\psi\rangle_c \otimes |\psi\rangle_k = &\cos\left(\frac{\arctan(20(t-5))}{2} + \frac{\pi}{4}\right) |0\rangle_c \otimes |0\rangle_k + \\ &\sin\left(\frac{\arctan(20(t-5))}{2} + \frac{\pi}{4}\right) |1\rangle_c \otimes |1\rangle_k \end{aligned} \tag{9}$$

[7] Owing to the scarcity of academic research on cat starvation a reasonable sounding estimate is used.

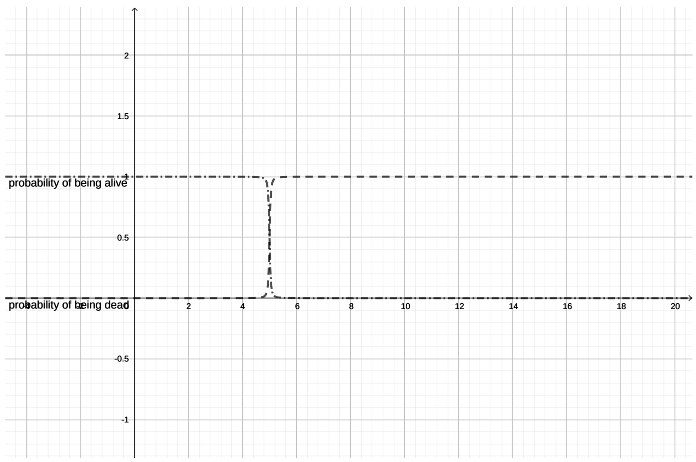

Figure 2: The probabilities of being alive and dead as a function of time as per equation 9.

The valve is opened on the 5th day with a probability of $\frac{1}{2}$ by tossing a fair coin to decide the outcome. The process in conducted in a random and fair manner. First consider what the probabilities would look like using classical analysis.

$$\begin{aligned} \boxed{0}_k &\leftrightarrow \frac{1}{2}\cos\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)^2+\frac{1}{2}\cos\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)^2 \\ \boxed{1}_k &\leftrightarrow \frac{1}{2}\cos\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)^2+\frac{1}{2}\cos\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)^2 \end{aligned} \tag{10}$$

This is severely underdefined. While the probabilities make sense it is very difficult to track the changes in the probability. It would be more clear to use a tree diagram to track the changes in the probability but this article attempts a different approach. Consider $|\psi\rangle_v$ to be the state of the valve. Then the combined state of the cat, knowledge and valve can be written as:

$$\begin{aligned} &|\psi\rangle_c \otimes |\psi\rangle_k \otimes |\psi\rangle_v = \\ &\frac{1}{\sqrt{2}}\left(\cos\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)|0\rangle_c\otimes|0\rangle_k+\sin\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)|1\rangle_c\otimes|1\rangle_k\right)\otimes|0\rangle_v \\ &\frac{1}{\sqrt{2}}\left(\cos\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)|0\rangle_c\otimes|0\rangle_k+\sin\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)|1\rangle_c\otimes|1\rangle_k\right)\otimes|1\rangle_v \end{aligned} \tag{11}$$

This state simultaneously captures all the different entanglements between the cat, knowledge and valve states.

Consider measuring the valve qubit, the states remaining would be:

$$\begin{aligned}\cos\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)|0\rangle_c\otimes|0\rangle_k+\sin\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)|1\rangle_c\otimes|1\rangle_k \leftrightarrow \frac{1}{2},\\ \cos\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)|0\rangle_c\otimes|0\rangle_k+\sin\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)|1\rangle_c\otimes|1\rangle_k \leftrightarrow \frac{1}{2}.\end{aligned} \tag{12}$$

This is just how the problem was built, there is a $\frac{1}{2}$ probability that the valve is opened and an equal probability that it wasn't.

Consider instead consulting somebody who possesses knowledge:

$$\begin{aligned}&|0\rangle_c\otimes\left(\frac{\cos\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)|0\rangle_v+\cos\left(\frac{\arctan(20)}{2}+\frac{\pi}{4}\right)|1\rangle_v}{\sqrt{\cos^2\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)+\cos^2\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)}}\right)\\ &\leftrightarrow\frac{1}{2}\left(\cos^2\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)+\cos^2\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)\right)\\ &|1\rangle_c\otimes\left(\frac{\sin\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)|0\rangle_v+\sin\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)|1\rangle_v}{\sqrt{\sin^2\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)+\sin^2\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)}}\right)\\ &\leftrightarrow\frac{1}{2}\left(\sin^2\left(\frac{\arctan(t-15)}{2}+\frac{\pi}{4}\right)+\sin^2\left(\frac{\arctan(20(t-5))}{2}+\frac{\pi}{4}\right)\right).\end{aligned} \tag{13}$$

This also captures the state if knowledge is possessed. As knowing something renders it true by definition, measuring $|0\rangle_k$ would force $|0\rangle_c$. This correlation is automatically captured by the state. The resulting state also contains information about the various probabilities of the valve being opened or closed.

Conclusion

The purpose of this paper was to show that quantum states naturally capture the idea of knowledge. Especially the idea of entanglement which gives naturally deals with the idea that knowledge of something cannot be false.

References

[1] Steup, Matthias and Neta, Ram. *Epistemology*. The Stanford Encyclopedia of Philosophy (Spring 2020 Edition), Edward N. Zalta (ed.). `<https://plato.stanford.edu/archives/spr2020/entries/epistemology/>`

[2] Nielsen, Michael A and Chuang, Isaac L. *Quantum Computation and Quantum Information: 10th Anniversary Edition*. Cambridge University Press, 2011.