Quantum Nature of Classical Information

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Properties such as entanglement and superposition are generally considered distinctly quantum. This paper attempts to demonstrate situations in which classical information systems show such properties. The paper starts with a brief introduction to the qubit and some of its properties. A quantum mechanical description of classical information, using qubits, is then developed wherein parallels between classical information and quantum states is drawn. This description is used to demonstrate how some classical systems show quantum behaviour. Finally, the popular chinese whisper game is analysed using perturbation theory. The paper concludes with a discussion on the classical limit of quantum information.

At length and time scales that correspond to common human experience, intuition is guided by classical physics[1]. This is because classical physics successfully explains most of the phenomena experienced at these scales, albeit with some arbitrary underlying assumptions. This shall henceforth be referred to as the *classical domain*. On the other hand, at length and time scales corresponding to atoms and nuclei, quantum theory successfully explains most of the phenomena, with some arbitrary underlying assumptions nonetheless, and this shall be referred to as the *quantum domain*. Classical behaviour sometimes seems to violate the principles of quantum mechanics, the paper starts with an attempt to explain these differences. On the other hand, quite surprisingly, some classical behaviour still possess quantum mechanical nuances and this shall be discussed next. The perspective of information is adopted.

I. INTRODUCTION TO THE QUBIT

Common sense dictates ideal classical information systems to possess certain properties. A few of the obvious ones are listed below.

- Objectivity: Inspired by [1], objectivity is the property by which different measurements of the classical information yields consistent results[2]. Consistency could be over different observers or over different times.
- Fixed Basis: Measurements of classical information yield results that are distinct, are mutually exclusive and belong to fixed categories. There exists nothing between a and b; this property is related to discreteness given below.
- Discreteness: While this is a property that is neither unique to nor possessed by all classical systems, most encodings of classical information is inherently discrete. A discrete alphabet is used to encode the information.

Many quantum information systems are also discrete, in that the basis states form a countable set. But they differ from classical information systems in the other two aspects. Firstly, different observers may obtain different outcomes. This happens, for example, if the quantum state is a superposition state. In this case, each basis state only has a certain probability of being observed. It is not possible to predict the outcome a priori. Because of this, objectivity is lost. There cannot be a consensus on the basis state that the system is in[3]. There is another way in which these quantum states may be inconsistently measured; and that corresponds to a "rotation" of the measuring apparatus. The measurement could be done in a different basis and that would affect the result; this again is non-classical.

Consider the example of a one dimensional quantum harmonic oscillator[3, p. 283][4]. The hamiltonian of the oscillator is:

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, \text{ with solutions,} \tag{1a}$$

$$\psi_0(x), \psi_1(x), \psi_2(x)..., \text{ satisfying,} \tag{1b}$$

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2\psi_n(x)}{\mathrm{d}x^2} + \frac{1}{2}m\omega^2 x^2 \psi_n(x) = E_n\psi_n(x). \tag{1c}$$

The $\psi's$ form the energy eigenstates of the system. Choosing any two states of the system, $\psi_0(x)$ and $\psi_1(x)$

[1] Classical physics referring to newtonian mechanics, classical thermodynamics, classical electromagnetism and so on. In this paper, relativistic effects are ignored in both, the classical and quantum domains.

[2] This is not to say that the same results would be obtained each time. In case one measures the *classical position* of a particle, one might get a different position at different points in time. However, the positions would be consistent with the velocity of the particle. That is what is meant by consistent.

[3] In this case, it is assumed that the quantum information system is a machine that consistently produces arbitrarily many instances of a certain general *quantum* state. It is possible to probe the state using techniques such as weak measurement and then obtain the state as a superposition[2]. The point here is to see that the behaviour differs from that of classical information.

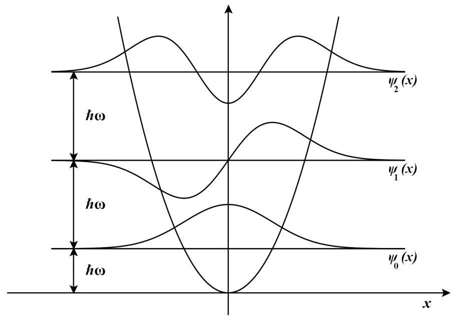

Figure 1. The Harmonic oscillator potential and wavefunctions.

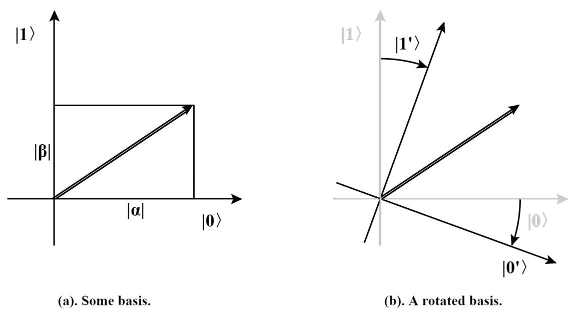

Figure 2. (a) Measurement in a basis. (b) Rotating the apparatus of measurement.

for convenience, and restricting the system to stay in these states, one gets a qubit. It is much more convenient to express these wavefunctions in the dirac notation and this shall be used henceforth. The states are thus,

$$\begin{aligned} |0\rangle &= \psi_0(x), \text{ and,} \\ |1\rangle &= \psi_1(x). \end{aligned} \tag{2}$$

These states differ from classical information basis states as they can be in a superposition state,

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,. \tag{3}$$

If an observer were to measure the state $|\psi\rangle$, they obtain state $|0\rangle$ with a probability of $|\alpha|^2$ and state $|1\rangle$ with a probability of $|\beta|^2$. It is possible that two observers measure something different given the same quantum state and, thus, measurement is not consistent[4]. Finally, the states can be measured by "rotating" the apparatus and this introduces further subtleties, see figure 2.

[4] It is again emphasised that the quantum information system does not produce just one instance of the state but rather produces arbitrarily many instances. In this sense, the quantum information produces the *quantum states* consistently, but that does not imply that the measured results would be consistent.

II. A QUANTUM DESCRIPTION OF CLASSICAL INFORMATION

If quantum mechanics is the correct description of matter, classical behaviour should be obtained as a limit of the quantum. However, as classical behaviour differs significantly from the quantum it is not obvious how the limit is obtained. This section attempts to use qubits to build a classical state; ad-hoc conditions are imposed on the qubits in order for classical behaviour to emerge. It further explores these ideas in the purview of quantum decoherence and einselection [5].

Classical behaviour is characterised by highly consistent, highly repeatable measurement outcomes. An attempt is made to obtain these classical states from qubits. An arbitrary qubit is represented in a state such as,

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle\,. \tag{4}$$

Here, $|0\rangle$ and $|1\rangle$ represent the basis states that the qubit is measured against. The basis states represent something physical such as the direction of magnetic field or the splitting of beams in an interferometer apparatus and are necessary while physically describing the qubit. As many quantum systems are infinite dimensional and also contain a large number of particles it would be impossibly cumbersome to describe them in the physical basis. In order to describe classical states it makes more sense to talk about a "logical" basis. The logical states encode whether or not the qubit is rotated in the correct direction with respect to obtaining the classical measurement outcomes[5].

This is best described using an example. Consider a tiny patch of paper in a book, on which is written 'a'. In this case the molecules of ink and the molecules of the paper form the physical part of the classical system. The letter 'a' is recognised by looking at those points in which the molecules of ink are absorbed and hence the positions of all the ink molecules, in the patch of paper, could be considered as the state space of the system. If one looks at a particular molecule of ink, only certain positions within the lattice of paper molecules would correspond to consistent representations of 'a'. Even though the Hilbert space is not two dimensional in this case, one could divide it into positions corresponding to consistent representations and those not. In this way, the state-space could be divided into,

$$|0_L\rangle \leftrightarrow \text{states(locations) corresponding to 'a'} \tag{5a}$$

$$\left|\bar{0}_L\right\rangle \leftrightarrow \text{states(locations) not corresponding to 'a'} \tag{5b}$$

[5] See, for example [3, p. 283] cited before. In this case the article goes on further do describe the logical states $|00_L\rangle$, $|01_L\rangle$, $|10_L\rangle$ and $|11_L\rangle$. These states are logical states in that they encode what values the qubit logically refer to, irrespective of their particular physical characteristics

Thus, each molecule of ink is in a state such as,

$$|\psi_L\rangle = \alpha\,|0_L\rangle + \beta\,\left|\bar{0}_L.\right\rangle \tag{6}$$

This discussion was for one instance of a representation of 'a'. In fact the logical state would remain unaltered if the entire text were shifted slightly to the left or to the right or if it were slightly expanded or contracted. Thus, there are many physical configurations that lead to a consistent logical state. This is one factor that adds to the resilience of classical states. In order to carry forward this discussion it is assumed that there exists the ideal 'a' whose physical state perfectly captures the essence of 'a'ness. The logical basis of the ideal 'a' is then used to describe any other instance. The $|0_L\rangle$ and $\left|\bar{0}_L\right\rangle$ refer to these basis states. Any instance of 'a' is not simply $|0_L\rangle^{\otimes N}$ but is a more complicated state, it would look like a mixed state with a certain probability distribution over states close to the ideal 'a'.

Before the 'a' is written, the molecules of paper and the molecules of ink are in a random, uncoordinated state. It is then written down and a coordinated 'a' state is formed; the interaction with the environment occurs when the state is being written. One could ask what caused the 'a' to exist in the first place, or why it it written the way it is. The 'a' written on a patch of paper is not a stand-alone system, its formation and existence is shared with every other 'a', and with several other interacting systems. There is a certain component of 'a'ness in any of its written instances. This is inevitably linked to the environment, it is in the writing of the 'a' that there is an interaction in the environment and where the component of 'a'ness gets infused in it. One could thus be write the state of the 'a' as,

$$\left|\psi_{'a'}\right\rangle = \left|\psi_{'a'}\right\rangle_P \otimes \left|\psi_{'a'}\right\rangle_L, \tag{7}$$

where the $|\psi_{'a'}\rangle_P$ corresponds to the physical part of the 'a', where the molecules are placed and so on, as discussed in a prededing paragraph, and the $|\psi_{'a'}\rangle_L$ corresponds to the logical part, that is guided by the interaction with the environment. The composite state $|\psi_{'a'}\rangle$ is intimately connected to both and the two subsystems are indispensable for the whole state. While the state of the $|\psi\rangle_P$ could be a mixed state; it is assumed that in the enlarged Hilbert space that it lives, $|\psi\rangle_{'a'}$ is indeed a pure state. The state $|\psi_{'a'}\rangle$ could thus be a superposition of many states of varying distance from the ideal 'a' state[6]. It is proposed that the composite state would look like a gaussian curve in the enlarged Hilbert space, figure 3. The motivation is that the errors tend to be normally distributed; a justification of this is provided in section IV.

The interaction with the environment stays on even after the 'a' is written;, the environment continues to

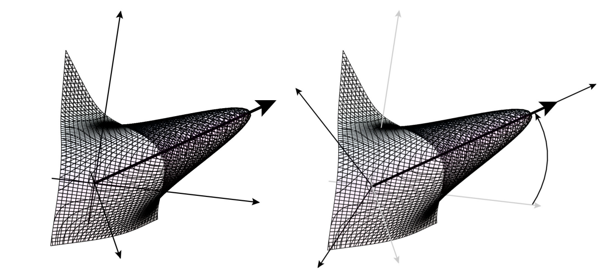

Figure 3. (a) Physical states making up the basis. The *classical* state pointing somewhere in the state space. (b) Rotating the basis to point along the classical state.

influence the logical state of the system, this is discussed later. Not only does the environment guide the formation of the state but so also does it guide its measurement. Any observer wishing to *read* the 'a' is guided by the environment in doing so. In a sense, the axes of measurement are *rotated* to correspond to the logical basis, figure 3(b).

To be more specific about the interaction with the environment it is assumed that there exists an 'a' gas; the part of the environment that influences the 'a' systems. This 'a' gas has an existence extended over the earth in space and over centuries in time. It is a sort of memetic state that tries to keep repeating itself. There is competition between the different versions of the 'a' and there is a quantum darwinism that einselects the states more suitable to the environment[1]. If in no other way, the 'a' state at least influences the environment in adding itself to the 'a' gas; thus, not only does the environment influence the 'a' state but so also does the 'a' system influence the environment[7].

$$\begin{aligned}&(|\phi\rangle_P \otimes |\phi'\rangle_L) \otimes \left|\psi_{'a'}\right\rangle_{env} \rightarrow \\ &(\left|\psi_{'a'}\right\rangle_P \otimes \left|\psi_{'a'}\right\rangle_L) \otimes \left|\psi'_{'a'}\right\rangle_{env}.\end{aligned} \tag{9}$$

Here the $|\phi\rangle$ represent random, uncoordinated states. And $|.\rangle_{env}$ represents the 'a' gas of the environment. There is similarly, a 'b' gas and a 'c' gas so on. These gases are governed by grammar and morphology[6], much like the 'a' gas guides the physical state of a written instance. These form even larger states that interact with the letter gases, they are grammar states and word states; only certain strings of letters make sense and only certain combinations of words make sense. These states in turn guided by meaning states and beauty states and so on.

[6] The ideal 'a' state here includes the physical and logical parts.

[7] This could also explain the ability of classical states to be copied,

$$\left|\psi_{'a'}\right\rangle \otimes |\psi_u\rangle \otimes |\phi_e\rangle \rightarrow \left|\psi_{'a'}\right\rangle \otimes \left|\psi_{'a'}\right\rangle \otimes |\phi'_e\rangle \tag{8}$$

Here, $\left|\psi_{'a'}\right\rangle$ is the classical 'a' states, $|\psi_u\rangle$ is an uncoordinated random state and $|\phi\rangle$ are environment states. The environment enables copying.

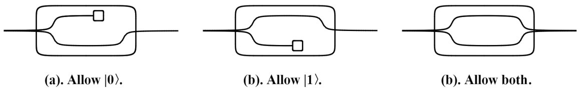

Figure 4. Schematic Stern-Gerlach apparatus. (a) Apparatus turned to only allow $|0\rangle$ states (b) Apparatus turned to allow only $|1\rangle$ states (a) Apparatus turned to allow all states.

There is a hierarchy in which the higher states guide the lower states.

In order to continue the discussion to classical systems that show quantum effects it would be helpful to declare a few quantum states that will be used. For our discussion it is enough to stick to the morpheme level. A morpheme is the minimum unit of meaning[6, p. 40]; for the purpose of the paper it could be described as a state that conveys something definitive. Some examples of morphemes are $|motorola\rangle$, $|milk\rangle$ and $|breakfast\rangle$. These all have specific meanings and the state codifies the specific meaning that these morphemes carry. Some other examples could be $|count\rangle$ and $|key\rangle$. But these morphemes are under specified. *count* could have multiple meanings and so could *key*, that is they have the same physical state but different logical states. How does one decide what meaning to use? The meaning usually becomes clear through context. If one wrote, "The count of monte cristo", the text takes on that value of $|count\rangle$. If on the other hand one wrote, "I lost count of the cards", the text takes on a different $|count\rangle$ value. The surrounding words cause the collapse to a particular $|count\rangle$ state! This is one of the ways in which the environment continues to influence a written state once it is written.

One could be a lot more specific while writing down the states. The notation $|count\rangle_{"count"}$ is used in place of the previous $|"count"\rangle_P \otimes |count\rangle_L$[8]. It refers to the specific $|count\rangle$ state that a particular written instance, *"count"* takes on. Thus, $|\psi\rangle_{"count"}$ collapses to a particular $|count\rangle$ state when surrounded by extra information. If, for example,

$$|0\rangle \leftrightarrow \text{count of monte cristo} \tag{10a}$$
$$|1\rangle \leftrightarrow \text{count of cards,} \tag{10b}$$

then, $|0\rangle_{"count"}$ refers to "count" as in "count of monte cristo" and $|1\rangle_{"count"}$ refers to "count" as in "the count of cards". "count" here might refer not just to a written state but also a spoken state, a digital copy or any other physical representation[9].

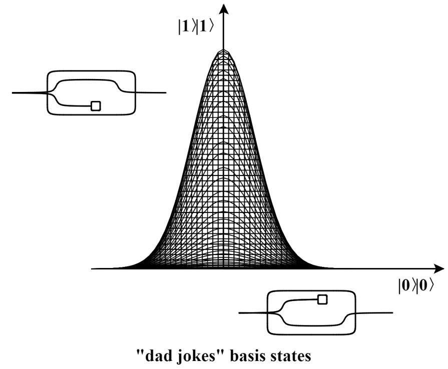

Figure 5. The states corresponding to "dad jokes" and the Stern Gerlach apparatus used to measure them.

It is interesting to look at the rotations guided by the higher order language states. In order to motivate this, it is assumed that the collapse of a state happens because of a Stern-Gerlach apparatus[7]. With respect to our example, the environment is acting as a Stern-Gerlach apparatus, guiding the axes as well as the specific value that the state collapses to. Figure 4 clarifies the issue.

III. QUANTUM BEHAVIOUR OF CLASSICAL INFORMATION SYSTEMS

It is hard to see quantum behaviour in the *classical states.* These states are *too classical* to behave quantumly. One must rather look at higher order language states such as grammar and meaning to get a clear picture of the quantum behaviour. This shall be demonstrated using an example.

Consider the phrase, "dad jokes". It could be interpreted in at least two different ways,

"dad jokes" - As in the noun "dad-jokes", the kinds of jokes dads tend to make. In this case "dad" is an adjective-noun and "jokes" is a noun. They are both half the noun "dad jokes". This could be denoted as state $|0\rangle$.

"dad jokes" - As in the noun - verb "dad jokes", the statement that dad makes jokes. In this case, "dad" is a noun and "jokes" is a verb. This could be denoted as state $|1\rangle$.

$|0\rangle$ and $|1\rangle$ are morpheme states. They are not exact quantum states but rather codify the *classical* states and guide their interpretation. They could be seen as the logical qubits for the *classical* states. Let the state space be restricted to the two morpheme states, see figure 5. In that case, the phrase "dad jokes" could correspondingly

[8] The single quotation marks used for 'a' were to make it stand out. Here the double quotation marks used for "count" represent a written instance of the system, in other words the physical part of the composite system.

[9] Observe how the morpheme states dictate the axes used to measure the "count" *classical* states. Higher order states like morphemes dictate the rotations of lower order *classical*, physical, states. The morpheme states are themselves dictated by even higher order language states such as spelling and grammar and meaning.

be in two different states,

$$|\psi\rangle_{"dad\ jokes"} = |0\rangle_{"dad"} \otimes |0\rangle_{"jokes"}\ , \text{ or,} \tag{11a}$$
$$|\psi\rangle_{"dad\ jokes"} = |1\rangle_{"dad"} \otimes |1\rangle_{"jokes"} \tag{11b}$$

When there is a source of ”dad jokes” phrases it is always assumed to be in one of the two morpheme states. The question is whether the state of such a phrase is a superposition state or a mixture state a prioi. It is clear that a state such as $|\psi\rangle_{"dad\ jokes",P}$, the physical representation of the information such as an instantiation on a piece of paper, is a classical state and thus a measurement would not change its state. On the other hand it is not clear whether the morpheme states, that is the logical states behave classically or quantumly. Beginning with a classical assumption, it is proposed that the state is in a mixture state such as,

$$|\psi\rangle_{"dad\ jokes"} = \begin{cases} |0\rangle_{"dad"} \otimes |0\rangle_{"jokes"}\ , & \text{probability p} \\ |1\rangle_{"dad"} \otimes |1\rangle_{"jokes"}\ , & \text{probability 1-p.} \end{cases} \tag{12}$$

This means that the state is along the basis, it only takes on either $|0\rangle_{"dad"} \otimes |0\rangle_{"jokes"}$ or $|1\rangle_{"dad"} \otimes |1\rangle_{"jokes"}$. And thus, when the phrase is given some context, the meaning becomes immediately apparent.

When one says,

”I love telling dad jokes.”

the state collapses[10] to $|0\rangle_{"dad"} \otimes |0\rangle_{"jokes"}$. This could be interpreted as a restriction of the measurement space to only one of the alternatives, see figure 6. So far, so good. If however, one proceeds to read further and comes across,

”I love telling dad jokes. He especially likes knock knock jokes”,

one is forced to choose the other alternative, $|1\rangle_{"dad"} \otimes |1\rangle_{"jokes"}$. If the state was in a mixture state and was restricted to only one alternative, it does not make sense to change that alternative later. It does not even make sense from a classical information perspective because in that case it seems to violate the objectivity conditions[11]!

To make matters worse, the sentence could further read,

”I love telling dad jokes. He especially likes knock knock jokes, but Matt enjoys the occasional dad joke too.”.

[10] Of course as a classical state it must have always been in that state.

[11] One might say that the state was always in a consistent state and not fully revealed when the sentence was only partially read. Well, it might be that the page is torn off after that phrase and so somebody may not get to read the complete sentence. In that case they would be *stuck* with the incomplete, apparently wrong collapse of the state!

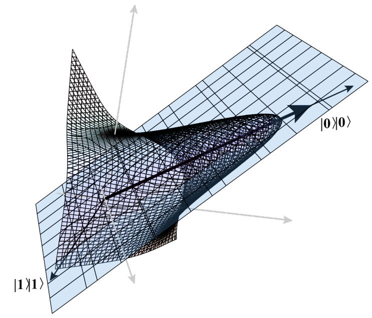

Figure 6. ”Collapse” of a classical state.

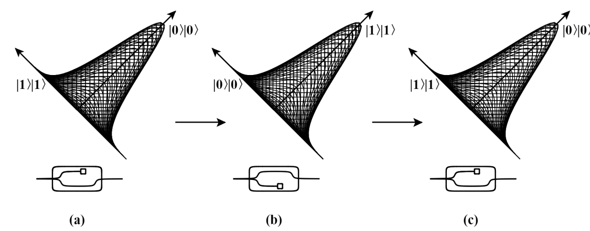

Figure 7. Rotation of the morpheme measurement axes to correspond to the classical state.

Now the state is back to being $|0\rangle_{"dad"} \otimes |0\rangle_{"jokes"}$! It must, thus, be that the morpheme state is in a superposition state of $|0\rangle_{"dad"} \otimes |0\rangle_{"jokes"}$ and $|1\rangle_{"dad"} \otimes |1\rangle_{"jokes"}$. A physical *classical* state, such as ”dad jokes” written on a piece of paper is unlikely to undergo such a change. This is because the states are too classical, they are very well designed. But even if the state was in a superposition such as $\alpha\,|0\rangle_{"dad"} \otimes |0\rangle_{"jokes"} + \beta\,|1\rangle_{"dad"} \otimes |1\rangle_{"jokes"}$, that does not answer the question. Once the state has settled to one of the basis states, it cannot turn into the other. If instead, one imagines the morpheme axes to turn to correspond to the *classical* state, it does not violate any of our requirements. The classical state remains unaffected by the rotation of the axes and the state can still be said to collapse to the required morpheme state. Thus, measurements in these systems correspond to rotations of the axes of measurement, see figures 6 and 7. Not only are classical states affected by morpheme states but so are morpheme state affected by classical states! Linear algebra saves the picture once again. All of this could be done with classical mechanics, what role does quantum mechanics play in this interaction?

One might wonder, could the morpheme ever be in a state such as $|0\rangle_{"dad"} \otimes |1\rangle_{"jokes"}$? Again, the environment does not allow that! It does not make sense to form a phrase out of a half noun and a verb! One could say that the axes cannot rotate in that direction, the environment prevents it. This is reminiscent of an entangled state[3, p. 95] $\alpha\,|0\rangle \otimes |0\rangle + \beta\,|1\rangle \otimes |1\rangle$. The state of any one of the morpheme fixes the state of the other.

Now consider, the set of statements,

”I love telling dad jokes!

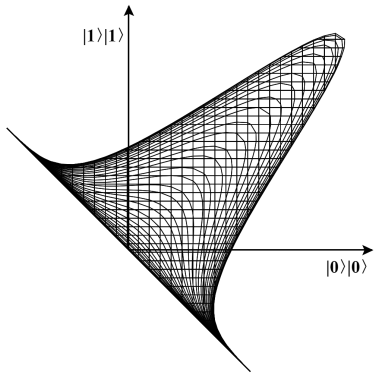

Figure 8. Rotation of the morpheme measurement axes by $\frac{\pi}{4}$

Figure 9. True or False?

He doesn't enjoy them so much though."

In what state is the "dad jokes" phrase now? Neither is it in the state $|0\rangle_{\text{"}dad\text{"}} \otimes |0\rangle_{\text{"}jokes\text{"}}$ nor is it in the state $|1\rangle_{\text{"}dad\text{"}} \otimes |1\rangle_{\text{"}jokes\text{"}}$. It seems to be in the state $\frac{1}{\sqrt{2}}|0\rangle_{\text{"}dad\text{"}} \otimes |0\rangle_{\text{"}jokes\text{"}} + \frac{1}{\sqrt{2}}|0\rangle_{\text{"}dad\text{"}} \otimes |0\rangle_{\text{"}jokes\text{"}}$! A superposition of morpheme states. The joke only makes sense if both morpheme meanings can simultaneously be realised. The axes have been turned by $\frac{\pi}{4}$! See figure 8.

Humour is one of the higher order language skills where such phenomena are observed. One should have no difficulty imagining such a superposition state for emotional states, philosophical quotes, moral questions and so on. In fact for some of these higher level states one could imagine a continuum between the states. For examples, the moral value of something can be imagined to be on a spectrum between good and bad. What's more these states evolve with time and are different in different places. So the morality wavefunction seems to have a variation over space and time. These states seem to be quite common!

What is surprising is that some of the more physical, lower order *classical states* also show such behaviour. The "Laurel-Yanny" debate is a recent example of something like that in the case of speech[8]. Again, one of the prerequisites is a closeness in physical space between the two states, that is, closeness of their classical states. In this case, it can be seen that the signals are close in frequency space[9]. See figure 9 for an example involving written text.

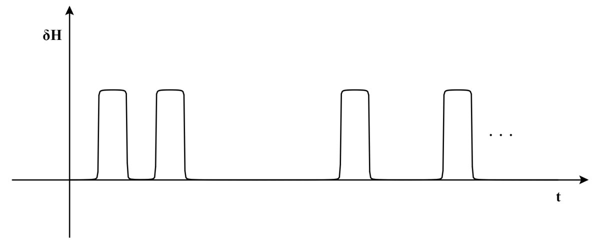

Figure 10. The perturbing hamiltonian has a constant amplitude and acts periodically over time.

IV. PERTURBATION THEORY

Consider again the 'a' state of section II. Before being written and after being written, the physical state $\left|\psi_{'a'}\right\rangle_P$ is only minimally affected by the environment; there may be changes to the logical state. It is while the 'a' is being written that the environment interacts with the state causing its einselection and this is the perturbation that causes the 'a' state to form. Consider once again the equation regarding environmental selection, equation 9; the hamiltonian driving this transition is a perturbation hamiltonian $\delta H(t)$ whose time dependence is such as to exist for only a small amount of time, wherein the 'a' is being written down, see figure 10.

Assume that the *ideal* 'a' state, $\left|\psi_{'a'}\right\rangle_P = |0_L\rangle^{\otimes N}$, is produced immediately after the hamiltonian acts, it then takes some amount of time before the state settles down; during this time the classical state *spreads* over the state space into a gaussian distribution, figure 3. The gaussian behaviour is seen by considering the states to spread in a random walk. A molecule in a state $|0_L\rangle$ would transition to a state $\left|\overline{0}_L\right\rangle$ with a probability p. As each molecule approximately has a certain fixed probability to change its state, the number of particles that possess the $\left|\overline{0}_L\right\rangle$ state would be binomially distributed with a mean Np and variance $Np(1-p)$. For sufficiently large N the normal distribution is approached. This idea is motivated from [10, p. 588].

In order to calculate the probability of transition one could look at the random walk of an ink particle. Assume that it is written on a patch of paper $1cm^2$. It takes about $1s$ for the ink to dry and the actual lines are about $0.5mm$ thick. The distance traversed by the molecule in a random walk has a mean 0 and a variance $R^2 = 2Dt$[7, 10], where D is the diffusion coefficient and t is the time of about a second. One gets $R \simeq 2.2 \times 10^{-5}m$ for a sucrose molecule[11, p.286]. In order for the molecule to change state from $|0_L\rangle$ to $\left|\overline{0}_L\right\rangle$ it would have to travel a distance of at least $0.5mm = 510^{-4}m$. The length required to travel is around 22.4 standard deviations. The probability of that occurring is about 10^{-111}! If the area of $1cm^2$ is broken into squares of $0.5mm$ one gets a grid of 400 segments. Assuming about a fifth of the space is occupied by the ink, one gets the number of possible states to be $\binom{400}{80}$ which is about 10^{85}. Even the immense number of states cannot out-power the extremely low probability of occurrence.

Consider a perturbing hamiltonian that has a transition amplitude of $\sqrt{p}$ leads to a probability p of changing state and E is its energy. This hamiltonian simply reassigns states with a different probability.

$$\delta H = E \begin{pmatrix} \sqrt{1-p} & \sqrt{p} \\ \sqrt{p} & \sqrt{1-p} \end{pmatrix} \tag{13}$$

Equation (5.3.35) of [12, Ch. 5] states that the probability of a transition to the continuum is given by equation,

$$P_{f \leftarrow i}(t) = \frac{2\pi}{\hbar} |\delta H_{fi}|^2 \rho(E_f) t. \tag{14}$$

In this case δH_{fi} is $E\sqrt{p}$, the transition amplitude. $\rho(E_f)$ is the density of states at the final energy and thus, $E\rho(E_f)$ equals the number of states, about 10^{85}. The probability of transition is,

$$P_{f \leftarrow i}(t) = \frac{2\pi}{\hbar} ENpt. \tag{15}$$

Substituting $\frac{3}{2}k_B T$ for the average energy, k_B being the boltzmann constant and T the temperature, it is seen that the probability of transition is about 10^{-12}, quite negligible.

V. DISCUSSION

The emergence of classical behaviour from the quantum has been discussed at great length[1, 5]. It is seen that classical behaviour is quite distinct from quantum behaviour and that the environment plays a major role in the transition from classical to quantum.

However, it is also seen that there is some residual quantumness in even these classical states. Viewing the environment as a hierarchy of systems that influence the ones below them, it can be seen how the classical behaviour is einselected to generate the immense objectivity in these states. There is a quantum darwinism that selects the states which are most suitable to the environment.

The ideas about perturbation theory is developed only approximately. It is critical to carry out the calculation in much more detail as the solutions are very sensitive to the input. All in all, this paper just touches upon the ideas that are developed here and does not go into much detail. There seems to be immense scope for further research into this topic.

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