Supersymmetry

Topics in Quantum Field Theory

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Outline

Some Prerequisites

The basic notation is developed. The weyl spinor is an important component here.

Poincaré Group and its Decomposition

The poincaré group is the set of isometries on minkowskian spacetime. There are four translations P_μ, three rotations J_i and three lorentz boosts iK_i, $\mu \in \{0,1,2,3\}$, $i \in \{1,2,3\}$. They obey the following commutation relations

$$[J_i, J_j] = i\epsilon_{ijk}J_k \qquad [J_i, K_j] = i\epsilon_{ijk}K_k \qquad [K_i, K_j] = -i\epsilon_{ijk}J_k$$

$$[J_i, P_0] = 0 \quad [J_i, P_j] = i\epsilon_{ijk}P_k \quad [K_i, P_0] = -iP_i \quad [K_i, P_j] = -i\delta_{ij}P_0$$

For example,

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} \qquad iK_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Poincaré Group and its Decomposition

There is "mixing" between the generators of rotation and boosts. However, there is a basis in which no mixing takes place. Using $J_{\pm i} \equiv \frac{1}{2}(J_i \pm iK_i)$,

$$[J_{+i}, J_{+j}] = i\epsilon_{ijk}J_{+k} \qquad [J_{-i}, J_{-j}] = i\epsilon_{ijk}J_{-k} \qquad [J_{+i}, J_{-j}] = 0$$

The $SO(3,1)$ set of isometries is broken down into two sets of SU(2)!

For $j = 0, \frac{1}{2}, 1, \frac{3}{2}, ...$, SU(2) can have a $(2j+1)$ dimensional representation with the vector space consisting of elements labelled by $-j, -j+1, ...j-1, j$. This means that $SO(3,1)$ representations are labelled by pairs of indices (j^+, j^-) on spaces of dimensions $(2j^+ + 1)(2j^- + 1)$,

$$(0,0), (\frac{1}{2}, 0), (0, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}), ...$$

Weyl Spinors

Just having a look at dimensions, $(0,0)$ looks like a scalar and $(\frac{1}{2},\frac{1}{2})$ like a vector. But what about $(\frac{1}{2},0)$, for example?

Label the $2\frac{1}{2}+1$ objects as ψ_α, $\alpha \in \{1,2\}$. This means that J_{+i} acts as $\frac{1}{2}\sigma_i$ while J_{-i} acts as 0 on this space. Which, in turn, means that,

$$J_i = \frac{1}{2}\sigma_i \qquad iK_i = \frac{1}{2}\sigma_i$$

Similarly, labelling the 2 objects in $(0,\frac{1}{2})$ as $\bar{\chi}^{\dot{\alpha}}$ leads to J_{+i} acting as 0, J_{-i} acting as $\frac{1}{2}\sigma_i$, and,

$$J_i = \frac{1}{2}\sigma_i \qquad iK_i = -\frac{1}{2}\sigma_i$$

Weyl Spinors

The two component spinors ψ_α and $\bar{\chi}^{\dot{\alpha}}$ are Weyl spinors. Under a parity transformation, $x \to -x$ and $p \to -p$,

$$\implies J \to J \qquad K \to -K \qquad J_\pm \to J_\mp$$

A dirac spinor is,

$$\Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$$

and,

$$\vec{J} = \begin{pmatrix} \frac{1}{2}\vec{\sigma} & 0 \\ 0 & \frac{1}{2}\vec{\sigma} \end{pmatrix} \qquad i\vec{K} = \begin{pmatrix} \frac{1}{2}\vec{\sigma} & 0 \\ 0 & -\frac{1}{2}\vec{\sigma} \end{pmatrix}$$

Dotted Notation

The lorentz group $SO(3,1)$ splits into two pieces with generators,

$$\vec{J} + i\vec{K} \leftrightarrow ()^{()} \qquad\qquad \vec{J} - i\vec{K}\ \dot{()}^{\dot{()}}$$

each isomorphic to $SU(2)$. Dots help keep track of which subspace is being referred to. Using the weyl basis,

$$\gamma^0 = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \qquad \implies \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

with $\sigma^\mu = (\mathbb{1}, \vec{\sigma})$ and $\bar{\sigma}^\mu = (\mathbb{1}, -\vec{\sigma})$ indicating that indices on the σs are,

$$(\sigma^\mu)_{\alpha\dot{\alpha}} \text{ and } (\bar{\sigma}^\mu)^{\alpha\dot{\alpha}}$$

This also makes dimesnional sense as σ^μ is a 4-vector and must look like $(\frac{1}{2}, \frac{1}{2})$.

Dotted Notation

Under a lorentz transformation, a spinor changes as $\Psi \to e^{-\frac{i}{4}\omega_{\mu\nu}\Sigma^{\mu\nu}}\Psi$ with $\Sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$ generating the transformation. Defining,

$$\Sigma^{\mu\nu} = 2i \begin{pmatrix} \sigma^{\mu\nu} & 0 \\ 0 & \bar{\sigma}^{\mu\nu} \end{pmatrix}$$

$$\implies \sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu) \text{ and } \bar{\sigma}^{\mu\nu} = \frac{1}{4}(\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)$$

This indicates indexing as $(\sigma^{\mu\nu})_\alpha^\beta$ and $(\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}}$ and concretely,

$$\sigma^{0i} = -\bar{\sigma}^{0i} = -\frac{1}{2}\sigma^i \qquad \text{and} \qquad \sigma^{ij} = \bar{\sigma}^{ij} = -\frac{i}{2}\epsilon^{ijk}\sigma^k$$

Under an infinitesimal lorentz transformation,

$$\psi_\alpha \to (\mathbb{1} + \frac{1}{2}\omega_{\mu\nu}\sigma^{\mu\nu})_\alpha^\beta \psi_\beta \qquad \bar{\chi}^{\dot{\alpha}} \to (\mathbb{1} + \frac{1}{2}\omega_{\mu\nu}\bar{\sigma}^{\mu\nu})_{\dot{\beta}}^{\dot{\alpha}} \bar{\chi}^{\dot{\beta}}$$

behaving just as in non-relativistic quantum mechanics.

Dotted Notation

Whether indices are upstairs or downstairs are of consequence in the supersymmetry language and is related to charge conjugation. A charge conjugated field is defined by $\Psi^C \equiv C\bar{\Psi}^T$, where $\bar{\Psi} = \Psi^\dagger \gamma^0$ and $C^{-1}\gamma^\mu C = -(\gamma^\mu)^T$. In the weyl basis, one choice is,

$$C = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix} \quad \Longrightarrow \Psi^C = \begin{pmatrix} i\sigma_2 \bar{\chi}^* \\ -i\sigma_2 \psi^* \end{pmatrix} \quad \text{and } (\Psi^C)^C = \Psi$$

Choosing,

$$\varepsilon_{\alpha\beta} = (i\sigma_2)_{\alpha\beta} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \varepsilon^{\alpha\beta} = (-i\sigma_2)^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

indicates lowering and raising of indices as $\psi_\alpha = \varepsilon_{\alpha\beta}\psi^\beta$ and $\psi^\alpha = \varepsilon^{\alpha\beta}\psi_\beta$ and also that $\varepsilon_{\alpha\beta}\varepsilon^{\beta\gamma} = \delta_\alpha^\gamma$. Analogously, $\bar{\chi}^{\dot{\alpha}} = \varepsilon^{\dot{\alpha}\dot{\beta}}\bar{\chi}_{\dot{\beta}}$ and $\bar{\chi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\chi}^{\dot{\beta}}$ with a numerically identical ε.

Dotted Notation

Noting that $(i\sigma_2)\sigma_i^*(-i\sigma_2) = -\sigma_i$

$$\implies (i\sigma_2)(\sigma^\mu)^*(-i\sigma_2) = -\bar{\sigma}^\mu \text{ and } (i\sigma_2)(\sigma^{\mu\nu})^*(-i\sigma_2) = \bar{\sigma}^{\mu\nu}$$

Now defining, $\bar{\psi}_{\dot{\alpha}} \equiv (\psi_\alpha)^*$ and $\chi^\alpha \equiv (\bar{\chi}^{\dot{\alpha}})^*$, it is seen that,

$$\Psi^C = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix} \Longleftarrow \Psi = \begin{pmatrix} \psi_\alpha \\ \bar{\chi}^{\dot{\alpha}} \end{pmatrix}$$

Another way to see this is that complex conjugation puts a dot as $\vec{J} + i\vec{K} \to \vec{J} - i\vec{K}$. These rules indicate what quantities are invariant just as in general relativity. For example,

$$\eta^\alpha \to \eta'^\alpha = \varepsilon^{\alpha\beta}\eta'_\beta = \varepsilon^{\alpha\beta}(e^{\frac{1}{2}\omega\sigma})_\beta^\gamma\eta_\gamma = \varepsilon^{\alpha\beta}(e^{\frac{1}{2}\omega\sigma})_\beta^\gamma\varepsilon_{\gamma\rho}\eta^\rho = (e^{-\frac{1}{2}\omega\sigma^T})_\rho^\alpha\eta^\rho$$

$$\implies \eta\psi \equiv \eta^\alpha\psi_\alpha = \eta^T\psi \to \eta^T(e^{-\frac{1}{2}\omega\sigma^T})^T(e^{\frac{1}{2}\omega\sigma})\psi = \eta\psi$$

is a scalar and so is $\bar{\chi}\bar{\xi} \equiv \bar{\chi}_{\dot{\alpha}}\bar{\xi}^{\dot{\alpha}}$.

The Basic Idea

In this section an attempt is made to give a simplistic argument for supersymmetry foreshadowing the more formal approach in the next section.

Changing Bosons into Fermions!

Familiar operators change bosons into bosons and fermions into fermions. For example, the momentum operator,

$$P_\mu \left|boson\right\rangle = p_\mu \left|boson\right\rangle \text{ and } P_\mu \left|fermion\right\rangle = p_\mu \left|fermion\right\rangle$$

What is sought now is an operator Q that changes fermions to bosons and vice-versa,

$$\mathrm{Q} \left|boson\right\rangle = \left|fermion\right\rangle \text{ and } \mathrm{Q}^\dagger \left|fermion\right\rangle = \left|boson\right\rangle$$

Note that spin is no longer a good quantum number! Q is supercharge, generator of supersymmetry.

Wess Zumino

Further suppose that Q satisfies the algebra,

$$\{\mathrm{Q}, \mathrm{Q}^\dagger\} = 4E \qquad \{\mathrm{Q}^\dagger, \mathrm{Q}^\dagger\} = 0 \qquad \{\mathrm{Q}, \mathrm{Q}\} = 0$$

and that its action on a bosonic complex scalar field ϕ and a fermionic 2 component weyl spinor ψ is as follows,

$$\mathrm{Q}\phi = \psi \qquad \mathrm{Q}^\dagger\psi = i\sigma^\mu\partial_\mu\phi$$

The claim is that this leaves the Wess Zumino Lagrangian,

$$\mathcal{L}_{WZ} = \partial_\mu\phi\partial^\mu\phi^\dagger + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi$$

invariant under a supercharge transformation!

A More Sophisticated Take

Following Zee, a more formal approach is attempted.

What else can it be?

Much of the mathematics seems to follow smoothly just from notation. Consider again the supercharge Q_α. The $()_\alpha$ is a reminder that this object takes in a scalar field and returns a weyl spinor. Being a weyl spinor it must abide by their customs, $[J^{\mu\nu}, \mathrm{Q}_\alpha] = -i(\sigma^{\mu\nu})_\alpha^\beta \mathrm{Q}_\beta$. For what else can it be?

Similarly, $[J^{\mu\nu}, \bar{\mathrm{Q}}^{\dot{\alpha}}] = -i(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \mathrm{Q}^{\dot{\beta}}$. And if the supercharge doesn't change over spacetime, $[P^\mu, \mathrm{Q}_\alpha] = 0$.

What else can it be?

Again, just by looking at indices it can be seen that,

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu \tag{1}$$

as σ^μ is the only object with the right spinor indices and it must be contracted with a spacetime vector for lorentz invariance.

Similarly, $\{Q_\alpha, Q^\beta\} = c_1(\sigma^{\mu\nu})_\alpha^\beta J_{\mu\nu} + c_2\delta_\alpha^\beta$. Commuting with P^λ shows that c_1 must vanish. Now use, $Q_\gamma = \varepsilon_{\gamma\beta}Q^\beta$ to see that $\{Q_\alpha, Q_\gamma\} = c_2\varepsilon_{\alpha\gamma}$. As one side is symmetric and the other is antisymmetric, c_2 must be 0. This shows that,

$$\{Q_\alpha, Q_\beta\} = 0 = \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\}$$

Superspace

$\{\mathrm{Q}_\alpha, \bar{\mathrm{Q}}_{\dot\beta}\} = 2(\sigma^\mu)_{\alpha\dot\beta} P_\mu$ indicates that a supersymmetric transformation followed by its conjugate generates a spacetime translation.

As $P_\mu = i\frac{\partial}{\partial x^\mu}$, propose the existence of two superspace "coordinates", θ^α and $\bar\theta^{\dot\beta}$ such that Q and $\bar{\mathrm{Q}}$ are operators that generate translations in them, respectively. However, in order for their anticommutator to work out properly, their definitions have to be a bit more nuanced,

$$\mathrm{Q}_\alpha = \frac{\partial}{\partial\theta^\alpha} - i(\sigma^\mu)_{\alpha\dot\alpha}\bar\theta^{\dot\alpha}\partial_\mu \qquad \bar{\mathrm{Q}}_{\dot\beta} = -\frac{\partial}{\partial\bar\theta^{\dot\beta}} + i\theta^\beta(\sigma^\mu)_{\beta\dot\beta}\partial_\mu$$

Note, θ are grassmannian.

Superfield

A superfield lives is superspace, $\Phi(x^\mu, \theta^\alpha, \bar{\theta}^{\dot{\alpha}})$ such that an infinitesimal supersymmetry transformation is of the form,

$$\Phi \to \Phi' = (1 + i\xi^\alpha \mathrm{Q}_\alpha + i\bar{\xi}^{\dot{\alpha}} \bar{\mathrm{Q}}_{\dot{\alpha}})\Phi$$

with ξ^α and $\bar{\xi}^{\dot{\alpha}}$, two grassmannian parameters.

Consider the two "orthogonal" objects to the Q,

$$\mathrm{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu \qquad \bar{\mathrm{D}}_{\dot{\beta}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\beta}}\partial_\mu$$

which anticommute with them. If a field satisfies $\bar{\mathrm{D}}_{\dot{\beta}}\Phi = 0$, then so does the infinitesimally different field Φ'. Such a field is called a chiral superfield.

Superfield

Defining, $y^\mu = x^\mu + i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}$, it can be noticed that,

$$\bar{\mathrm{D}}_{\dot{\beta}} y^\nu = [-\frac{\partial}{\partial\bar{\theta}^{\dot{\beta}}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\beta}}\partial_\mu] y^\nu = i\theta^\alpha(\sigma^\mu)_{\alpha\dot{\beta}} - i\theta^\beta(\sigma^\mu)_{\beta\dot{\beta}} = 0$$

And so, any field such as $\Phi(y,\theta)$ is automatically chiral. As θ is a two component grassmannian object there can be at most two powers of θ,

$$\Longrightarrow \Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{2}$$

$$= \phi(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) \tag{3}$$

$$+ i\theta\sigma^\mu\bar{\theta}\partial_\mu\phi(x) - \frac{1}{2}\theta\sigma^\mu\bar{\theta}\theta\sigma^\nu\bar{\theta}\partial_\mu\partial_\nu\phi(x) + \sqrt{2}\theta i\theta\sigma^\mu\bar{\theta}\partial_\mu\psi(x) \tag{4}$$

where ϕ and F are scalar fields and ψ is a weyl field.

Superaction

Dimensional analysis used to find the superaction! P_μ has the dimension of mass, $[P_\mu] = 1$. Looking at equation (1) it is seen that, $[\mathrm{Q}] = [\bar{\mathrm{Q}}] = \frac{1}{2}$, which in turn shows that $[\theta] = [\bar{\theta}] = -\frac{1}{2}$.

Positing, $[\Phi] = [\phi] = 1$ as they are scalar fields leads to, $[\psi] = \frac{3}{2}$ (equation (2)) and interestingly, $[F] = 2$! F is already strange as ψ and ϕ are the required superpartners, where did it creep in?

An infinitesimal deviation $\delta F \sim \xi, \bar{\xi}$ with $[\xi] = [\bar{\xi}] = -\frac{1}{2}$, indicating that it can only couple to $\partial_\mu \psi$ ($\partial_\mu \bar{\psi}$ being excluded as a result of not being part of Φ). Using again the "what else can it be" approach,

$$\delta F \sim \partial_\mu \psi^\alpha (\sigma^\mu)_{\alpha\dot{\alpha}} \bar{\xi}^{\dot{\alpha}}$$

due to the requirements of lorentzian and weylian contractions. Note that F is a total derivative!

Superaction

Given any superfield Φ, if $[\Phi]_F$ is its $\theta\theta$ coefficient, then $\delta[\Phi]_F$ is a total divergence and $\int d^4x[\Phi]_F$ is invariant under supersymmetric transformations. Also as any $\Phi^k \equiv \Phi^k(y,\theta)$, all Φ^k are chiral fields. This indicates that the required supersymmetry invariant action is of the form,

$$\int d^4x[\frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3 + \ldots]_F = \int d^4x[W(\Phi)]_F$$

It is seen that the $\theta\theta$ terms are $[\Phi^2]_F = (2F\phi - \psi\psi)$,
$[\Phi^3]_F = 3(F\phi^2 - \phi\psi\psi)$ and so on.

Superaction

In order to get fermionic terms with derivatives, terms such as $\bar{\psi}$ need to be considered and, thus, $\Phi^\dagger$ is to be used. A term such as $\Phi^\dagger\Phi$ for which $V^\dagger = V$ is called a vector superfield. Again being grassmannian, the highest power terms are $\bar{\theta}\bar{\theta}\theta\theta$. Again, call $[V]_D$ the coefficient of $\bar{\theta}\bar{\theta}$ and use dimensional analysis.

If V has mass dimension n, $[V]_D$ has dimensions $n+2$. $\delta[V]_D \sim \xi, \bar{\xi}$ with the ξ coefficient having a dimension of $n+\frac{5}{2}$. This can only be the ∂_μ of a term with dimension $n+\frac{3}{2}$ in V, that is, coefficients of terms $\bar{\theta}\bar{\theta}\theta$ or $\bar{\theta}\theta\theta$ in V. This means that $\delta[V]_D$ is again a total divergence!

Superaction

Looking at equation, it is seen that $\int d^4x[\Phi^\dagger\Phi]_D$ contains the terms $\int d^4x\phi^\dagger\partial^2\phi$, $\int d^4x\partial\phi^\dagger\partial\phi$, $\int d^4x\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi$ and $\int d^4xF^\dagger F$.

$$\implies S = \int d^4x\{[\Phi^\dagger\Phi]_D - ([W(\Phi)]_F + h.c.)\}$$

With an explicit choice such as, $W(\Phi) = \frac{1}{2}m\Phi^2$,

$$S = \int d^4x\{\partial\phi^\dagger\partial\phi + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + F^\dagger F - (mF\phi - \frac{1}{2}m\psi\psi + h.c.)\}$$

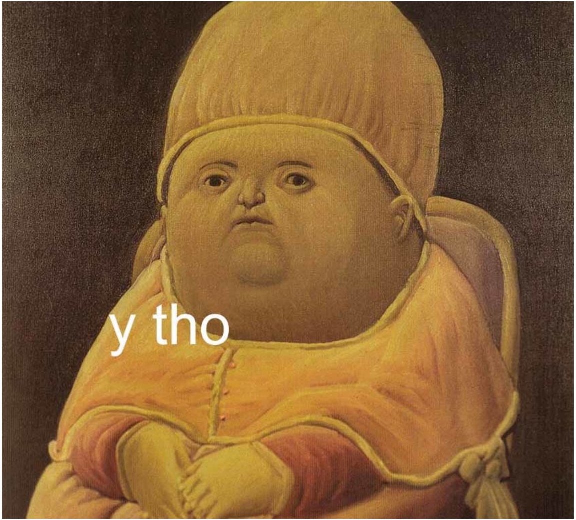
The field F has no kinetic terms and therefore does not contribute to the dynamics. In fact, it can be integrated out by $\int \mathcal{D}F^\dagger\mathcal{D}Fe^{iS}$. Collecting the terms depending on F,

$$F^\dagger F - mF\phi - m\phi^\dagger F^\dagger = |F - m\phi^\dagger|^2 - |m\phi|^2$$

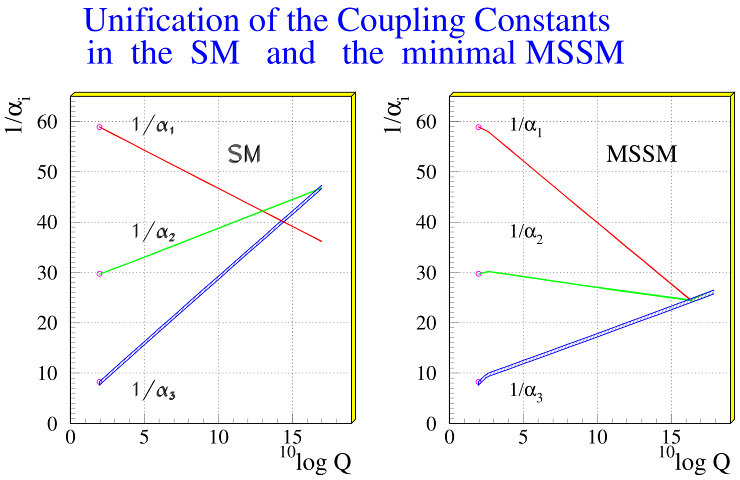
$$\implies S = \int d^4x\{\partial\phi^\dagger\partial\phi + i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi - |m\phi|^2 - (\frac{1}{2}m\psi\psi + h.c.)\}$$

y tho?

All this begs the question



Unification



There is a unification of coupling constants in the supersymmetric model as opposed to the standard model. α_1, α_2 and α_3 refer to the electromagnetic, weak and strong forces respectively.

The Hierarchy Problem

Particles have masses proportional to the higgs mass. Interactions with bosons increase the higgs mass whereas interactions with fermions decrease its mass. In the standard model, these are unrelated and there is a huge discrepancy between the expected mass of the higgs particles and what is observed.

Supersymmetry requires that each boson is paired with a fermion thereby exactly cancelling the contributions. This may explain why the observed mass of the higgs particle is orders of magnitude lower than expected.

Coleman-Mandula and Beyond the Standard Model

The coleman-mandula theorem states[4] that for every quantum field theory satisfying,

- finite number of particles below any mass
- any two particle state undergoes some reaction at all energies
- the amplitude for two body scattering is an analytic function of scattering angle at almost all energies

there can only be a lie group symmetry which is a direct product of the poincaré group and any internal symmetry groups. This is a problem for string theories and such.

The resolution is, of course, proposing a lie super algebra! A $\mathbb{Z}_2$ graded algebra with a super skew-symmetry and a super jacobi identity.

Note: The actual problem is a lot more nuanced.

Energy Divergence

Owing to the fact bosons obey a commutation relationship, whereas fermions obey an anticommutation relationship, their vacuum energy contributions are of opposite signs. Supersymmetry can overcome the issue of diverging vacuum contribution by coupling a boson to each fermion and vice-versa.

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Thanks to all the listeners. Any questions?